



K22P 1603

Reg. No. :

Name :

I Semester M.Sc. Degree (C.B.S.S. – Reg./Sup./Imp.)
Examination, October 2022
(2019 Admission Onwards)
MATHEMATICS
MAT1C03 : Real Analysis

Time : 3 Hours

Max. Marks : 80

PART – A

Answer **four** questions from this Part. **Each** question carries **4** marks :

1. Prove that compact subset of metric spaces are closed.
2. Give an example of an open cover of the segment $(0, 1)$ which has no finite subcover.
3. If $f(x) = |x|^3$. Show that $f'(0)$ does not exist.
4. Using L'Hospital's rule, evaluate $\lim_{x \rightarrow 0} \frac{1 - \cos 2x}{x^2}$.
5. Show that the polynomial $f(x) = x^5 + x^4 + x^3 + x + 1$ is of bounded variation on $[0, 1]$.
6. If $\int_a^b f d\alpha = 0$ for every f which is monotonic on $[a, b]$. Prove that α must be a constant on $[a, b]$.

PART – B

Answer **any four** questions from this Part without omitting **any** Unit. **Each** question carries **16** marks :

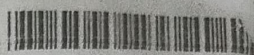
Unit – I

7. a) Prove the following :

- i) For any collection $\{G_\alpha\}$ of open sets, $\bigcup_\alpha G_\alpha$ is open.
 - ii) For any collection $\{F_\alpha\}$ of closed sets, $\bigcap_\alpha F_\alpha$ is closed.
 - iii) For any collection $G_1, G_2, \dots, G_n$ of open sets, $\bigcap_1^n G_i$ is open.
 - iv) For any collection $F_1, F_2, \dots, F_n$ of closed sets, $\bigcup_1^n F_i$ is closed.
- b) Show that there exist a perfect set in $\mathbb{R}^1$ which contain no segment.

P.T.O.





8. a) Let A be the set of all sequences whose elements are the digit 0 and 1. Prove that A is uncountable.
 b) Prove that countable union of countable set is countable.
9. a) Prove the following : Suppose $Y \subset X$. A subset E of Y is open relative to Y if and only if $E = G \cap Y$ for some open subset G of X .
 b) Let A, B are two subsets of a metric space X . Prove that
- i) $\overline{(A \cup B)} = \bar{A} \cup \bar{B}$
 ii) $\overline{(A \cap B)} \subseteq \bar{A} \cap \bar{B}$.

Unit – II

10. a) State and prove the Generalized Mean Value Theorem.
 b) Suppose $f'(x) > 0$ in (a, b) . Prove that f is strictly increasing in (a, b) and let g be its inverse function. Prove that g is differentiable and that $g'(f(x)) = \frac{1}{f'(x)}$, $a < x < b$.
 c) Prove the following : If f is monotonic on $[a, b]$ and if α is continuous on $[a, b]$, then $f \in R(\alpha)$.
11. a) If $f, g \in R(\alpha)$ on $[a, b]$. Prove that (i) $f g \in R(\alpha)$ (ii) $|f| \in R(\alpha)$ and $\left| \int_a^b f d\alpha \right| \leq \int_a^b |f| d\alpha$.
 b) Prove the following :
 i) If $f(x) \leq g(x)$ on $[a, b]$, then $\int_a^b f d\alpha \leq \int_a^b g d\alpha$.
 ii) If $f \in R(\alpha)$ on $[a, b]$ and if $a < c < b$, then $f \in R(\alpha)$ on $[a, c]$ and $[c, b]$.
12. a) State and prove L' Hospital's Rule.
 b) Suppose a and c are real numbers, $c > 0$, and f is defined on $[-1, 1]$ by $f(x) = x^a \sin(|x|^{-c})$, $x \neq 0$, and $f(0) = 0$. Prove the following :
 i) f is continuous if and only if $a > 0$.
 ii) $f'(0)$ exist if and only if $a > 1$.
 iii) f' is continuous if and only if $a > 1 + c$.
 iv) $f''(0)$ exist if and only if $a > 2 + c$.



Unit – III

3. a) State and prove the fundamental theorem of calculus.
b) Let f be of bounded variation on $[a, c]$ and $[c, b]$. Prove that $V_f(a, b) = V_f(a, c) + V_f(c, b)$.
14. a) Assume that f, g are each of bounded variation on $[a, b]$. Prove that $V_{f \pm g} \leq V_f \pm V_g$ and $V_{fg} \leq AV_f + BV_g$ for some $A, B \geq 0$.
b) Show that the function $f(x) = x \sin\left(\frac{1}{x}\right)$, $x \neq 0$ and $f(0) = 0$ is not of bounded variation on $\left[0, \frac{2}{\pi}\right]$.
15. a) Given that $f = e^{2\pi it}$ if $t \in [0, 1]$ and $f = e^{2\pi it}$ if $t \in [0, 2]$. Prove that the length of g is twice as that of f .
b) Prove that : Two paths f and g in $\mathbb{R}^n$ are equivalent if and only if they have the same graph.
c) Examine whether the function $f(x) = x^2 \cos\left(\frac{1}{x}\right)$, $x \neq 0$, $f(0) = 0$ is of bounded variation on $[0, 1]$.

